Against Second-Order Logic

Against Second-Order Logic: Quine and Beyond.

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Abstract: Is second-order logic logic? Famously Quine argued second-order logic wasn't logic but his arguments have been the subject of influential criticisms. In the early sections of this paper, I develop a deeper perspective upon Quine's philosophy of logic by exploring his positive conception of what logic is for and hence what logic is. Seen from this perspective, I argue that many of the criticisms of his case against second-order logic miss their mark. Then, in the later sections, I go beyond Quine to develop a novel case that quantification into polyadic predicate position, understood as requiring quantifiers to range over relations, isn't intelligible.

Key words: second-order logic, sets, relations, completeness, semi-decidability, quantification, Quine, Boolos, Shapiro

1. *Introduction*

Can higher-order logic provide a logical framework for metaphysics or philosophy of mathematics? Not if higher-order logic isn't logic.

Quine argued that higher-order logic isn't logic. Nowadays his arguments are cursorily dismissed. But they deserve a fair hearing and we can still learn from them and be inspired by them. Quine had a profound sense of the universal significance of logic for our cognitive economy—to how we make sense of ourselves, one another and the world. Because higher-order logic does not have that kind of significance for us, higher-order logic was not logic for him.

That's not to say that every argument Quine offered was a good one. It can't be denied that some of his well-known arguments for refusing to allow quantification into predicate position really don't pass muster—not unless one is already convinced of their conclusion. But, relatively speaking, the shortcomings of these particular arguments aren't that significant in the grand scheme of things. There are deeper considerations, more challenging to the idea that higher-order logic is logic, to be discerned in Quine's philosophy.

Here my aim is to capture the animating spirit of Quine's philosophy of logic—his positive characterisation of what logic is and should be—in virtue of which Quine deemed first-order but not second-order logic, to be logic. I begin by developing the exegetical case that Quine conceived logic to be obvious in a behavioural sense and it was because second-order logic isn't obvious that he denied second-order systems the status of logic. From this perspective some of the most influential criticisms of Quine on second-order logic can be addressed. In subsequent sections I pass beyond Quine to develop a novel case against quantifying into predicate position which doesn't rely upon Quine's philosophy of logic but is compatible with it. If successful, this argument casts doubt upon the very idea of higher-order quantification understood as quantification over relations, a case which would have been congenial to Quine, a 'neo-Quinean' argument.

The termini of these two lines of reflection are different—that, respectively, (*i*) higher-order logic, even if intelligible, lacks the positive features in terms of which Quine characterised logic, and (*ii*) that higher-order quantification, understood as quantification over relations, is unintelligible. But they share the consequence that higher-order logic, understood in terms of relations, cannot provide a logico-structural framework for metaphysics or the philosophy of mathematics.

2. *A Concession and a Charitable Hypothesis*

It's a characteristic feature of higher-order logics that they permit quantification into predicate position. If predicate quantification is understood in terms of *substitution* of predicates for variables, then a higher order existential quantification is counted as true if some instance, which results from substituting a predicate for the variable, is true. But this makes the meaning of the existential quantifier dependent upon the availability of predicates in a language. To avoid such expressive limitations, higher-order quantifiers are usually understood as having a 'range'. The range consists of entities of some appropriate sort—whether sets or properties or relations—entities which are eligible for assignment as values to predicate variables even if no predicate determines them. A higher-order existential quantification is then counted as true if the open sentence after the quantifier is satisfied by an entity belonging to its range upon some assignment of values to variables.

It is against higher-order quantification, conceived in such range and entity-invoking terms, that Quine inveighed. To develop Quine's case, I put his *Philosophy of Logic* (1970) centre stage, because it's the work which provides the fullest elaboration of some of his most deeply considered and distinctive views.

It has to be conceded that two of Quine's best-known arguments for refusing to allow quantification into predicate position, arguments to be found in his *Philosophy of Logic*, just aren't effective—not if presented as self-standing arguments. For the first argument Quine directs us to the 'ordinary quantifications': '(∃x)(*x* walks)', '(∃*x*)(*x* is prime)'. Here the bound variables occur in name positions and what are said to walk or to be prime are things that could be named by names in those positions. From this Quine surmised, 'To put the predicate letter '*F*' in a quantifier, then, is to treat predicate positions suddenly as name positions, and hence to treat predicates as names of entities of some sort' (1970: 66-7). This is to treat predicates as names but predicates aren't names, so predicate positions aren't eligible for quantification on pain of confusing two quite distinct types of expression. Quine's second argument for refusing to quantify into predicate position relies upon the premise that predicates have attributes as their intensions or meanings (or would have if there were attributes) and sets as their extensions but are names of neither. From this premise Quine reasoned, 'Variables eligible for quantification therefore do not belong in predicate positions. They belong in name positions' (1970: 67).

The first argument isn't effective because, as Boolos pointed out, it doesn't follow from the fact that *some* quantifications, the 'ordinary' ones, only involve quantification into name position that *all* quantifications are just like the ordinary ones—that there aren't 'extraordinary' quantifications which also involve quantification into predicate position. So, *contra* Quine, there's no risk of confusing predicates with names just by allowing quantification into predicate position. The second argument isn't effective either because, as Boolos also pointed out, Quine's premise, that predicates have attributes as their intensions and sets as their extensions, doesn't preclude predicate variables standing indefinitely to a range of extensions as predicates stand definitely to their extensions (1975: 511). The more general problem with both arguments is that they presuppose what they are meant to show—that only quantification into name position is permitted, *ergo* not predicate position.

Is that the end of the story, Quine caught begging the question? It's certainly true that these arguments shouldn't convince anyone starting out cold. But if someone was already persuaded that only quantification into name position is permitted, these arguments might serve the different purpose of spelling out a consequence of what they'd already come to believe. At any rate the charitable interpretative hypothesis is that Quine offered these arguments at a stage when he was already convinced that only name positions are open to quantification, so the reasons that really convinced him must be found intellectually upstream. What lies upstream for Quine, as we'll see, is his dual conception of what logic is and what it does for us.

3. *Traits of Logic*.

What, for Quine, is logic? Quine took logical enquiry to investigate the determining links between sentences whereby it's settled that if one sentence is true it's logically implied another is. Nevertheless, for convenience, he subordinated the notion of logical implication to logical truth by a chain of definitions: p logically implies q iff p is logically incompatible with $\neg q$; p and $\neg q$ are logically incompatible iff the conjunction $p \& \neg q$ is logically false; $p \& \neg q$ is logically false iff $\neg(p \& \neg q)$ is logically true. So our question becomes, what, for Quine, is logical truth?

In *Philosophy of Logic* Quine identified three 'traits' of logical truth:

(1) *Obviousness*: 'the remarkable obviousness or potential obviousness of logical truth';

(2) *Topic Neutrality*: 'logic favours no distinctive portion of the lexicon and neither does it favour one subdomain of values of variables over another';

(3) *Universal Applicability*: 'the ubiquity of the use of logic. It is handmaiden of all the sciences, including mathematics' (1970: 98-9).[1]

It's in terms of these traits that Quine described the relationship between logic, mathematics and science. *Universal applicability* makes logic akin to mathematics, *obviousness* and *topic neutrality* makes them different. Mathematics is akin to logic because it has application throughout the sciences. It's also this wide applicability that separates logic and mathematics on the one hand from the sciences on the other. But this doesn't mean that mathematics is topic neutral the way logic is. This is because, Quine explains, 'Mathematics has its favoured lexicon, unlike logic, and its distinctively relevant values of variables' (1970: 98). Mathematics relies upon its lexicon to distinguish and quantify over different kinds of numbers and sets. It favours these subdomains of values of variables over subdomains favoured by other sciences—unlike logic which has no distinctive lexicon. Nor is mathematics obvious in the way that logic is. Quine conceived of every logical truth as 'either obvious as it stands or can be reached from obvious truths by a sequence of individually obvious steps' (1970: 82-3). Quine didn't mean by 'obvious' anything like 'incorrigible' or 'self-justifying'. Rather Quine used 'obvious' in what he described as 'an ordinary behavioural sense, with no epistemological overtones'. Something is obvious to a community only if 'everyone, nearly enough, will unhesitatingly assent to it' (1970: 92). The rules of a system are likewise obvious if everyone, nearly enough, will unhesitatingly assent to the transformations they license. By contrast, 'Mathematics, surely, even elementary number theory, is not potentially obvious throughout' (1970: 98). But large portions of mathematics, including elementary number theory, are accessible from unobvious beginnings—for example, mathematical induction—by taking what are mostly logical steps. Hence, Quine concluded, 'what stands forth is less a kinship of

[1] Quine identified a fourth trait, 'our tendency, in generalizing over [logical truths], to resort to semantic ascent' (1970: 102). Quine argued that semantic ascent is required to generalise over logical truths because quantifying within the object language is limited to quantification into name position. Since this presupposes quantification into predicate position isn't available, I set this trait aside.

mathematics to logic than the extreme efficacy of logic as handmaiden to mathematics'.

Quine didn't conceive the traits of logical truth he'd identified as just a jumble of independent features. Quine defined a logical truth in substitutional terms as '*a sentence that cannot be turned false by substituting for lexicon, even under the supplementation of lexical resources*' (1970: 60). Substituting for lexicon means substituting the lexical constituents of a sentence for other lexical elements belonging to the same grammatical categories. Hence a sentence is a logical truth if all sentences that share its grammatical structure are true—because sentences have the same grammatical structure when one can be converted into another by lexical substitutions.[2] It's in terms of this substitutional definition that Quine explained why the traits of logical truth go naturally together.

Take *Obviousness*. According to Quine, speakers of a language may vary in their knowledge of the lexicon but not the grammar. This is because whoever deviates from the grammar has either failed to master the language or speaks a different dialect, whereas a difference in lexicon reflects merely a difference in the lexical resources speakers have acquired. *Ergo* the logical truths, being tied by definition 'to the grammar and not to the lexicon will be among the truths on which all speakers are likeliest to agree (if we disregard examples that engender confusion through sheer complexity)' (1970: 102).

Now *Topic Neutrality*. In order to have a special subject matter, logical truths would have to draw upon a favoured portion of the lexicon to describe it. This would mean logical truths would have to consist of distinctive lexical elements. But logical truths, being tied by definition to the grammar and not the lexicon, lack distinctive lexical elements. Likewise, *Universal Applicability,* is 'explained by the invariance of logical truth under lexical substitutions' (1970: 102). Since speakers of a language have a grammar in common, however their lexicons may differ, they have logic in common, because a truth is logical only if

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[2] Quine (1970: 58) conceives this more abstract definition of logical truth as complementary to the familiar definition of logical truth in terms of an inventory of logical particles: logical truths are truths that stay true of under substitution of their constituent words and phrases, provided that the logical words '=', 'or', 'not', 'if-then', 'everything, 'something, etc., stay undisturbed.

all sentences which share its grammatical structure are true regardless of how their constitutive lexical elements differ.

We don't need to endorse Quine's substitutional definition to appreciate that Quine's traits of logical truth reflect valuable roles in our cognitive economy. It's a familiar thought that we cannot make sense of disagreement except against a background of agreement. But neither can we make sense of someone reasoning differently from us except against a background of shared reasoning. It's only because some truths and principles of reasoning are embodied in shared and unhesitating behaviour, hence obvious in Quine's sense, that we are able to communicate and understand one another. If someone were to routinely stall or turn another way when we don't flinch, we would be at a loss to find in their behaviour a pattern which we could recognise as an exercise of rationality. A shared logic must already be embedded in the structure of a purposive life in order for there to be intelligibility. And we bring the intellectual flights of science and mathematics home by subjecting them to the rigours of the logic to which we all unhesitatingly assent. Then we all have the prospect of a deeper and intersubjective understanding of science and mathematics whereby their axioms are laid bare and their theorems and observational consequences seen to follow.

Recognising the significance of logic for our cognitive economy is one thing but identifying which system performs the role of logic is another. Quine didn't think this could be done *a priori*. For him it was an empirical matter which system is embodied in collective patterns of unhesitating assent—from everyone, nearly enough—and he identified the first-order system of quantifiers and truth functions as the system which, as a matter of fact, does so. For this reason, Quine held that first-order logic is built into our canons of translation. Translation should 'Save the obvious' and since first-order logic is peculiarly obvious, we shouldn't represent speakers of another language as contradicting first-order logic (1970: 83).

A reading of the final section of 'Two Dogmas of Empiricism' (1951) encourages the impression that Quine took logic as consisting of highly theoretical statements akin to the highly theoretical statements of physics and likewise justified abductively. But Quine's mature view was that logic is obvious whereas theoretical physics isn't, and that our patterns of unhesitating behaviour aren't synchronised to the extent that they are because it's been established the system to which we adhere to be justified abductively—albeit Quine recognised that theory, perhaps even physical theory, did have a part to play in deciding how best to articulate the logic embedded in our behaviour. It's a further consequence of Quine's mature view that what's logic for us may change because another system may become embedded in our unhesitating behaviour over time—hence which system counts as logic may be different at different points in history.

Quine contrasts the first-order system of quantifiers and truth functions with set theory on the grounds that set theory doesn't perform the role of logic for us. That's because there are many different competing theories of sets and there is no consensus amongst mathematicians about whether one set theory is the correct theory of sets, never mind unhesitating agreement in the community at large. There is no consensus because there are no obvious solutions to the paradoxes of set theory which avoid contradiction whilst, for example, permitting the acknowledgment of infinite sets of different sizes. As Quine reflected, 'so far as is known, no consistent set theory is both adequate to the purposes envisaged for set theory and capable of substantiation by steps of obvious reasoning from obviously true principles' (1960a: 354). Really nothing that's distinctive about sets has been obvious since the discovery of the paradoxes. As a consequence, the canon of translation 'Save the obvious' doesn't behove crediting speakers of another language with a grasp of set theory.

Quine also described logic as 'ontologically innocent' (1953b: 114). A system that has a distinctive ontology of its own, must have a favoured portion of the lexicon to describe the values of its variables. But a system that relies upon a favoured portion of the lexicon to describe the values of its variables can't be obvious to speakers with different backgrounds because difference in background is reflected in difference in lexicon. So whatever system performs the role of logic cannot have its own ontology. By contrast, set theory isn't ontologically innocent. Set theory favours '∈' as a portion of the lexicon. It's a genuine predicate which cannot be explained away as a *façon de parler* and set theory makes use of this predicate to describe the classes which figure as the values of its bound variables.

4. *Is Second-Order Logic Set Theory in Disguise?*

In *Philosophy of Logic*, Quine famously argued that second-order logic is 'set theory in sheep's clothing' and because what's underneath the fleece is really set theory, he concluded second-order logic isn't logic.

Quine provisionally granted that the values of higher-order bound variables have a range. But what are the values of these variables? There seem to be two options: either they are attributes or they are sets. Quine dismissed attributes as a viable option on the grounds that they are inadequately individuated. Attributes can be different even though the same things exhibit them and Quine despaired of making sense of what else might be required for identifying them. By contrast, sets are well-individuated by the law of extensionality, which identifies sets whose members are the same. Hence Quine chose sets as the more credible option (1970: 67-8).

Quine's case that sets, unlike attributes are well-individuated, isn't watertight. The law of extensionality doesn't guarantee that sets are well-individuated, because sets are no better individuated than their members and their members may not be. But Quine had no need to dismiss attributes on grounds of inadequate individuation. If attributes are to be adequate for the purposes of higher-order logic then attributes need to be at least as abundant as sets. But the paradoxes are as much a problem for attributes when taken to be abundant as the paradoxes are for sets. Since there are many theories of attributes and there is no consensus about how, for example, Russell's paradox is to be avoided, theories of attributes are no more obvious than set theory. Because theories of attributes are ontologically committed to attributes, they're not ontologically innocent either. Since *obviousness* and ontological innocence are traits which matter to Quine so far as logic is concerned, nothing is lost by his focusing on sets because attributes are no better in these respects than sets.

Second-order logic, if it's logic, licenses existential quantification into predicate position. If so, it follows from the logical triviality (*i*) '$(\forall x)(Fx \leftrightarrow Fx)$' that (*ii*) '$(\exists G)(\forall x)(Gx \leftrightarrow Fx)$' is true too. Since there is a trivial logical truth of the form (*i*) for every first-order predicate in the language and the existential quantifier which features in sentences of the form (*ii*) ranges over sets, it also follows that for every such predicate there is a corresponding set *as a matter of logical truth*. Hence, Quine concluded in the 1st edition of *Philosophy of Logic*, set theory's 'staggering existential commitments' are forced upon us by licensing existential quantification into predicate position (1970: 68).

Quine was exaggerating because, as Boolos pointed out, second-order logic is only committed to the existence of the empty set. But this commitment is 'exceedingly modest' as Boolos put it (1975: 520). Keeping our current focus, (*i*) may be true even though nothing is *F*, in which case (*ii*) is only committed to the existence of the empty set. Moreover, it's worth remembering that the validity of first-order statements of the forms '$(\exists x)(Fx \vee \sim Fx)$' and '$(\forall x)Fx \supset (\exists x)(Fx)$' presuppose there being something in the universe, so even first-order logic has its existential commitments.

Shapiro also called Quine out for exaggerating. A second-order language will always be committed to more than the first-order language of which it is an extension—because second-order quantifiers range over all the subsets of the range of the first-order quantifiers (2012: 315-6). For example, second-order arithmetic is committed to sets of numbers, in addition to the numbers over which the bound variable of first-order arithmetic range. But this hardly compares to ZF's commitment to an unending sequence of infinite cardinals.

In the 2nd edition of *Philosophy of Logic* Quine toned down the rhetoric, claiming that only 'a fair bit of set theory has slipped in' when existential introduction for predicates is licensed (1986: 68). But, Shapiro reflects, this still leaves open 'just how much set theory has "slipped in" […] and it is not said what is bad about this. How much is a 'fair amount', and how much is too much?' (2012: 317). According to Shapiro, Quine couldn't answer these questions. That's because Shapiro attributes to Quine, based upon a reading of 'Two Dogmas', a strong form of epistemological holism whereby the sum total of our beliefs constitutes a seamless web, from which perspective mathematics blends into logic. Shapiro concludes, 'To wax Quinean, why should there be a sharp border separating mathematics from logic, especially the logic of mathematics?' (2012: 323).

From the deeper perspective upon Quine's philosophy of logic developed in preceding sections, it's clear that Boolos' and Shapiro's arguments aren't effective. First of all, it isn't obvious or potentially obvious that any set exists—admitting the existence of even one set is already a set too far for logic. It especially isn't obvious or potentially obvious that there is an empty set—not when a set is typically conceived as a 'many which can be thought of as one' (Cantor) even though the empty set has no members. So, *contra* Boolos, it's no consolation that second-order logic is only committed to the empty set—because that's already enough to show, by Quine's lights, that second-order logic isn't logic. Indeed, as Shapiro has himself emphasized, if second-order logic is logic then there is a second-order sentence which is a logical truth if and only if the continuum hypothesis is true and another second-order sentence which is logically true if and only if the continuum hypothesis is false.[3] But neither of these sentences is obvious or potentially obvious any more than the continuum hypothesis itself. As for the existential presuppositions of first-order logic, sometimes Quine justified them on grounds of technical convenience. But a better answer would have been that whilst it's not obvious there are sets, it's obvious there's something—because that's an assumption embodied in our behaviour.

Why should there be a sharp boundary separating mathematics from logic, especially the logic of mathematics? There should be a sharp boundary because the traits of logic demarcate a role for logic which isn't a role mathematics performs in our cognitive economy. It's because logic has the distinguishing trait of *obviousness* that logic has extreme efficacy as the handmaiden to mathematics, enabling us to see that large parts of mathematics whilst neither obvious nor potentially obvious are accessible by what are mostly obvious logical steps from unobvious beginnings—for example the axioms of Peano arithmetic. But if the logic of mathematics were itself imbued with mathematical content that is neither obvious nor potentially obvious then logic could not have the extreme efficacy it does.

Can this be squared with Quine's epistemological holism? Quine did indeed view empirical evidence as ultimately evidence for our whole system of beliefs, because no belief could be tested without presupposing others. But from

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[3] See Shapiro 1991: 105, 2012: 312.

the fact that our beliefs face the tribunal of experience *en bloc*, it doesn't follow that our beliefs form a seamless web. It doesn't follow from the empirical evidence being evidence for a whole system that the system itself lacks internal structure or that different parts don't have different ontological commitments.

In *Word & Object* (1960) Quine distanced himself from 'an excessive holism espoused in occasional passages of mine'—the passages in 'Two Dogmas' that suggests the view that logic blends into mathematics.[4] Later, in *Philosophy of Logic*, Quine described science as a 'interlocked' system 'including mathematics and logic as integral parts'; he admitted the boundaries between mathematics and logic could be obscured but 'I do not want thereby to suggest that the boundary is a minor one, or a vague one. On the contrary, I think it is important and worth clarifying' (1970: 72, 99). It's important, not least, because whereas mathematics presupposes sets, first-order logic, which performs the logic role for us, does not.

5. *Completeness and Decidability*

Quine's reasons for disfavouring second-order logic are often taken to include a requirement that logic should be complete, second-order logic being incomplete. But the incompleteness of second-order logic does not feature explicitly as a reason for his disfavouring it. So what was the significance of completeness for Quine? Completeness, I will argue, was significant for him only as a consequence of *obviousness* being a trait of logic.

Quine considered the first-order logic of the quantifiers '∀x' and '∃y' and truth functions, to be an integrated body of logical theory 'with bold and significant boundaries', as well as being free from paradox and a paragon of clarity, elegance and efficiency (1970: 90). Quine took one 'manifestation of the boldness of these boundaries' to be the fact that first order logic has a complete proof procedure for validity and one for inconsistency too (1970: 90-1). Quine did entertain the suspicion that this is too narrow a conception of logic because '∀*x*' and '∃*y*' might be usefully supplemented with branching quantifiers. But he put the suspicion aside on the grounds that the logic of branching quantifiers

[4] See Quine 1960b: 13.

does not admit of complete proof procedures for both validity and for inconsistency. By contrast, Quine held that the identity predicate belongs in logic because as well as being topic neutral and universally applicable, there is a complete proof procedure for '=', '$\forall x$' and '$\exists y$' (1970: 64).

Now Quine didn't consider completeness a constitutive trait of logic—it wasn't on his list of traits. It's also clear that *topic neutrality* and *universal applicability* are quite independent of completeness. But Quine did hint at a connection between completeness and *obviousness* when he remarked, 'Mathematics, surely even elementary number theory, is not potentially obvious throughout; it does not even admit of a complete proof procedure' (1970: 98). This implies that having a complete proof procedure for a system amounts to less than its being potentially obvious. So why mention it at all?

It's worth mentioning because without a complete proof procedure, a system doesn't even have the potential for being potentially obvious. Suppose a system S to be complete. Then if the rules of *S* are obvious, any logical truth *D* will be potentially obvious too because the completeness of *S* guarantees that *D* can be reached by a finite sequence of steps. Each step will be individually obvious because the rules of *S* are obvious. But it doesn't follow from the fact that *S* is complete that any logical truth expressible in *S actually is* obvious or potentially obvious. The rules of *S* may fail to be obvious in which case it won't follow that the logical truths expressible in *S* are potentially obvious even if *S* is complete. It will only follow from the completeness of *S* that they have the potential for being potentially obvious, *i.e.* potentially obvious *if* the rules of *S* are obvious. So completeness taken by itself cannot be what constitutes the obviousness or potential obviousness of logical truths.

What constitutes the obviousness of logical truths for Quine is the behavioural fact that everyone, nearly enough, will unhesitatingly assent to them. The rules of a system are obvious if everyone, nearly enough, will unhesitatingly assent to the transformations they license. Logic, for Quine, thus emerges from the uniform behaviour of a community of speakers—their uniform and unhesitating assent to certain truths and rules. Completeness doesn't get into the picture until speakers converge in their behaviour, finding certain truths and rules obvious. The completeness of a system, if its rules are already obvious to speakers, guarantees that the array of logical truths they potentially recognize will include all the logical truths, because they can all be reached by a sequence of obvious steps.

Boolos argued that 'Completeness cannot by itself be a sufficient reason for regarding the line between first- and second-order logic as the line between logic and mathematics' (1975: 523). He pointed out that there is a patchwork of similarities and differences between first and second-order logic. Second-order logic, unlike first-order logic, is indeed incomplete but monadic second-order logic, like monadic first-order logic, is decidable. It's true that polyadic second-order logic isn't decidable, *i.e.* a logic that allows a two or more place predicate to occur in quantified sentences, but polyadic first-order logic isn't decidable either. As Boolos reflected, 'There are decidable fragments of first-order logic, *e.g.*, monadic logic with identity, but decidability vanishes if even a single two-place letter is allowed in quantified sentences' (1975: 523).[5] Hence, his question, 'Why *completeness* rather than *decidability*' as a sufficient reason for distinguishing logic from mathematics?

Quine, I've argued, didn't take the completeness by itself to be a sufficient reason for regarding a system as logic because completeness only guarantees potential potential obviousness. So the short answer to Boolos is that completeness doesn't provide a sufficient reason for distinguishing logic from mathematics—it's obviousness and potential obviousness that matters. But there's a longer answer which builds upon the difference between decidability and completeness.

6. *Decidability and Semi-decidability*

In the 2nd edition of *Methods of Logic,* Quine reflected on the fact that first-order logic isn't decidable even though it is complete (1962: 190-1). The set of first-order logical truths isn't decidable because decidability requires an effective method for determining, in every case, *whether or not* a given formula belongs to it, but Church had established that no such method could be given for the set of first-order logical truths. Nevertheless, completeness tells us that every first-

[5] See further Boolos and Jeffrey 1974: 251.

order logical truth can be proved by the rules of the system. What's the relationship between completeness and decidability? From the completeness of first-order logic it follows that if a formula is a first-order logical truth then it can be proved valid. It doesn't follow, however, that if a formula is *not* a first-order logical truth then it can be proved not to be valid. This means that completeness only provides half of what decidability requires, namely an effective method covering both favourable and unfavourable cases. In other words, completeness only guarantees semi-decidability rather than decidability.[6]

What does that mean? If asked whether a given formula *D* belongs to the class of first-order logical truths, completeness guarantees there is an effective method which produces the answer 'yes' for *D* just in case it is a logical truth, *i.e.* *D* can be proved by the rules of the system. But if *D* isn't a logical truth then whilst the method won't produce the answer 'yes', it may continue indefinitely without producing the answer 'no'.

Quine held it to be a manifestation of the fact that the boundary between logic and mathematics had been crossed that first-order logic is semi-decidable whereas second-order logic is not, *i.e.* second-order logic is incomplete. Boolos took this position to be unmotivated because, he declared, decidability is 'every bit as significant a property' as semi-decidability (1975: 523). But what counts as a more or less significant property depends upon the context in which the properties are evaluated. And in *this* context, one in which Quine has identified *obviousness* as a trait of logic, lack of semi-decidability is an even more significant property than decidability. Because second-order logic isn't even semi-decidable, second-order logical truths cannot be potentially obvious in virtue of being guaranteed to be accessible by a finite number of obvious steps—as first-order logical truths are. In order for second-order logical truths to be accessible in this way, there would need to be an effective method for determining that a formula which is a second-order logical truth is a logical truth. But the incompleteness of second-order logic, its lack of semi-decidability, settles that no such method is to be had. It's not just that we can't cover the negative cases—that is, determine by an effective method that the formulas which aren't second-order logical truths,

[6] See Enderton 2001: 63.

aren't. It's that in second-order logic, we can't cover the positive cases either. But the semi-decidability of first-order logic guarantees that all first-order logical truths are potentially obvious (assuming the first-order rules to be obvious) because semi-decidability settles there is an effective method for determining that each first-order logical truth is logically true.

By contrast, decidability isn't a requirement of obviousness or potential obviousness because there doesn't need to be an effective method to determine that formulas which aren't logical truths aren't valid, in order for there to be an effective method to determine that formulas which are logical truths are valid. So long as we have the positive cases covered, we don't need to cover the negative ones too. Hence, in the context where obviousness or potential obviousness has been identified as a trait of logic, decidability really is a less significant property than completeness.

It may seem counterintuitive to say that decidability is less significant than semi-decidability because when we pick someone out as a reliable informant about *F*s, we often do so by forming a positive estimation of their capacity to settle not only whether something is *F* but also whether something isn't *F*. We are thereby reassured that their being reliable about *Fs* isn't a mere fluke—because they wouldn't have mistaken something which isn't *F* for something that is *F*. Part of the rationale for this practice is that typically the same underlying capacity is exercised when it is judged that something is *F* in the positive case as when it is judged something isn't *F* in the negative case. So if a subject fails to be reliable in the negative case that casts doubt upon their reliability in the positive case.[7] But whilst this way of thinking is appropriate with respect, for example, to perceptual knowledge, where typically the same perceptual capacities are exercised when a subject sees that something is *F* as when they see that something is not *F*, the semi-decidability of first-order logic demonstrates that this way of thinking cannot be straightforwardly carried across to the logical domain. By utilising an effective method for determining that the first-order logical truths are logical truths, a subject may be a reliable informant about the positive cases, even though there is no effective method that

[7] See Craig 1990: 58.

they could have employed for determining that formulas which aren't logical truths aren't.

What about Boolos' point that first-order monadic logic is decidable and so is second-order monadic logic? Quine identified *universal applicability* as a trait of logic alongside *obviousness*. But were decidability taken to be a sufficient reason for deeming a system logical then logic would be applicable only to relatively weak systems that are insufficient for the purposes of mathematics and science. Most mathematical and scientific theories rely on formulae whose constituent expressions include polyadic predicates and at least two binary arithmetic function signs—because most mathematical and scientific theories include (at least) Peano arithmetic.[8] Because they have this kind of grammatical and lexical complexity, it follows that most mathematical and scientific theories are undecidable. So if the impartial participation of logic in all the sciences is a trait of logic then decidability cannot be a criterion of logicality. So even though first-order and second-order monadic logic are decidable that doesn't make them logic—for the reason that they lack the wide range of applications characteristic of logic.

7. *Egalitarian versus Elitist Logic*

Quine portrays logic as egalitarian by nature—*obviousness* is a trait of logic and an integral part of what it means to be obvious is that logic is shared by everyone. Because second-order logic doesn't have the trait of *obviousness*, second-order logic fails to meet Quine's standards for egalitarian logic, regardless of whether second-order quantifiers range over sets or attributes.

Quine's vision of logic as egalitarian stands in stark contrast to the vision of logic as elite that, for example, Williamson puts forward in *Metaphysics as Modal Logic* (2013). Williamson presents his own higher-order logic as far from being obvious because the whole community might be undecided on such an abstruse issue (2013: 308-9). He interprets his logic as implying that everything

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[8] So whilst Pressburger arithmetic and Skolem arithmetic are decidable first-order theories with only one binary function symbol apiece, for addition and multiplication respectively, they are insufficient for Peano arithmetic which requires both addition and multiplication. See Quine 1962: 247-8.

exists necessarily and he doesn't think this is a conclusion that can be accessed by obvious steps made by just anyone. This is because Williamson conceives of logic, like natural science, as having an abductive methodology, where the choice of logic is guided by the assessment of the relative strengths and weakness of different logics. The virtues of his favoured logic, Williamson holds, are the very same theoretical virtues that are ascribed to appealing scientific theories, including fruitfulness, simplicity and elegance. Crucially, for Williamson, the assessment of the relative strengths of higher-order modal logic compared to other logics, just like the assessment of scientific theories, falls to the appropriate disciplinary experts. In this case the appropriate disciplinary experts are philosophers who have acquired the relevant logical and metaphysical skill sets through training and experience. It's upon their 'educated instinct' and 'good sense' which we must ultimately rely (2013: 428).

If, however, logic is elite rather than egalitarian, it cannot perform the roles Quine assigned to logic. If logic isn't obvious or potentially obvious then it cannot be a bridgehead for shared insight because it won't be something upon which we can all agree, except for the 'experts'. Nor if logic is elite can logic be the indispensable handmaiden for mathematics and science, a figure to illuminate our way—because on the elitist conception, logic itself will be no more obvious than the unobvious beginnings of mathematics and science.

Williamson does not claim for his elitist logic the advantages Quine claimed for his egalitarian one. Williamson's logic is intended as a vehicle for experts to frame ambitious metaphysical hypotheses of extreme generality, not help us to make sense of each other, and sense of the world, starting from a shared outlook. But it doesn't follow that Quine's and Williamson's logics are just ships passing in the night.

Quine may grant the epistemic possibility that one day at least some significant fragment of Williamson's higher-order modal logic might become obvious—although there is no special reason to expect this will happen anytime soon given that even the experts don't currently agree. But looking at the matter from the other side, Williamson's elitist logician cannot afford to forgo a place for Quine's egalitarian one. We need at least some shared logic to communicate and serve as a basis for translation and we understand scientific and mathematical theories better, the more elementary the logic we rely upon to analyse them. And the elite logician must grant that it is only from the humble beginnings of more elementary logic, which is continuous with the shared sense of logical consequence that we have as ordinary language users, that we can work our way up. Even elite logicians don't come in the world fully-armed.

8. *Quantifying into Predicate Position*

Quine's method relies upon both abstract reasoning and informed empirical conjecture. He asks what logic is for and locates the system he judges does that for us. The outcome relies upon an empirical hypothesis—that first-order logic corresponds, for the most part, to the patterns of our shared unhesitating behaviour. But that means Quine's views about the logic status of second-order logic, where they rely upon this hypothesis, are hostage to empirical fortune and it's a serious issue whether first-order logic corresponds, for the most part, to patterns of unhesitating behaviour. It may be that only something weaker than first-order logic is obvious. Or it may be that something stronger than first-order logic is.

Certainly Strawson, amongst others, have argued that there is evidence of higher-order reasoning in ordinary language—for example, when we reason from 'Tom does whatever William does' to 'if William hops then Tom hops', 'whatever' being understood as a higher-order quantifier figuring in action-predicate position.[9] But, on the other hand, such an inference may be understood as involving first-order quantification over action types and it's dubious whether ordinary speakers would assent unhesitatingly to the second-order logical truth that any two things have something in common ($\exists X\,(Xj \,\&\, Xm)$) or unhesitatingly deny the second-order logical falsehood that there are two things that have nothing in common ($\exists x \exists y \exists X\, \neg(Xx \,\&\, Xy)$).

These matters would be set in a different light if the very idea of higher-order quantification was shown to be incoherent—at least when conceived in terms of higher-order quantifiers having a range of properties and relations. What I'll now develop is an argument for the incoherence of higher-order

[9] See Strawson 1974/1997: 64-8 and Williamson 2013: 227.

quantification, conceived in terms of such a range. Quine didn't make this argument but it has roots in his philosophy. I offer it as a neo-Quinean argument which complements but doesn't require a thoroughgoing commitment to Quine's philosophy of logic. In this section I lay out the presuppositions of the argument. In the next I set out the argument itself.

The intelligibility of quantifying-in, when the quantifiers in question are understood as having a range, presupposes what I will call a Division of Semantic Labour, *DSL* for short. *DSL* is the principle that it must be possible to distinguish an expression in a sentence *S* whose role it is to identify a thing or pick a thing out from the rest of *S* whose complementary role is to say something about it, *i.e.* says something about it independently of how it was picked out.[10]

The case for *DSL* may be made in general terms. Consider the following more precise version of the principle. In order for a given position *p* in a construction *S* to be open to quantification, the expression *e* which occupies *p* must pick out something *x* whilst the remainder of *S* must discharge the complementary role of saying such-and-such about *x* independently of how *x* was picked out by *e*. In other words, it must be possible to use *S* minus *e* to say such-and-such about *x*.

The thinking behind this principle is closely related to what Quine described in "Reference and Modality" (1953c: 145) as the thinking behind the rule of existential generalisation itself*:* that if such-and-such is truly said of the item denoted by a given expression then such-and-such may also be truly said of something, *i.e.* the value of a bound variable. For suppose *DSL* doesn't hold. Then what *S* says about the denotation of *e* depends upon there being an occurrence of *e* occupying *p*. It follows that the existential generalisation of *S* that results from subtracting *e* from *p* and replacing it with a bound variable cannot say about the value of a bound variable, what *S* says about the denotation of *e*. So in order for the existential generalisation of *S* to say about something what *S* says about the denotation of *e*, *S* minus *e* must be a self-standing semantic unit—self-standing in

[10] For quantification into the positions of plural terms: it must be possible to distinguish expressions whose role it is to identify *some* things and the rest of the sentence which says something about *them*.

the sense that its significance doesn't shift or, worse, disintegrate upon *e*'s subtraction and replacement by a bound variable. *Ergo* DSL must hold.

This argument for *DSL* is further reinforced by the reflection that the truth-preserving characters of the rules of Existential Generalisation and Universal Instantiation are threatened if *DSL* fails to hold. For suppose that a predicate *F* fails to be a self-standing semantic unit, so meaning one thing in a singular context: *a* is *F* and another thing, or, indeed, nothing at all, in an existentially quantified context: something is *F* or everything is *F*. Then it cannot be guaranteed that if it's true that *a* is *F*, it's also true that something is *F*, or that if it's true that everything is *F* then it's also true that *a* is *F*.

It might be responded that if *F* is used to make a weaker claim when it occurs in an existentially quantified context than when it occurs in a singular context, then it will be guaranteed that if *a* is *F* is true then something is *F* is true too. But this only safeguards the truth preserving character of Existential Generalisation at the expense of jeopardising the truth-preserving character of Universal Instantiation. Because then it cannot be guaranteed that if everything is *F* then *a* is *F* because the instance is stronger than the quantified claim and it cannot be a guaranteed that if a weaker claim is true then a stronger one is. This problem might be overcome by claiming that a quantified construction in which *F* occurs makes a stronger claim than any instance in which it occurs. But this only safeguards the truth-preserving character of Universal Instantiation at the expense of jeopardising the truth-preserving character of Existential Generalisation.

The only plausible way out of these difficulties is to accept *what logicians have always assumed*, that where quantifying-in is permitted, *F* has a uniform significance regardless of whether it occurs in a quantified construction or an instance of one. But this is to endorse *DSL*—because what logicians have assumed presupposes that *F* functions as a self-standing semantic unit capable of surviving the extraction of a constant and its replacement with a bound variable, and the extraction of a bound variable and its replacement with a constant.

8. *Applying the Division of Semantic Labour*

Can *DSL* be applied to higher-order quantified constructions? Take a higher-order claim of the form,

(6) $\exists X\ aXb$

which arises from taking an atomic sentence of the form,

(7) aRb

and replacing the binary predicate '*R*' with a higher-order variable 'X' and binding it with a second-order quantifier. Now if (6) is to follow from (7) then what (7) says of the denotation of *R* must be what (6) says of some relation. But in order for this to be the case, (7) must include a free-standing semantic unit which recurs in (6), so that what (7) says of the denotation of *R* is what (6) says of some relation. In other words, DSL must hold.

Which, if any, expressions are candidates to be a free-standing semantic unit which recurs in (6) and (7)? The only plausible candidate is the one place, higher-order predicate '*aXb*'—one place because it has a single argument place capable of being filled by a relational predicate. In other words, to recognise the validity of the inference from (7) to (6), we need to recognise that (7) admits of the decomposition,

(9) $aXb + R$

and (6) of the decomposition,

(10) $aXb + \exists X$

where the decompositions make salient the recurrence of the higher-order predicate *aXb*. Of course, we don't *need* to understand (7) as featuring an occurrence of *aXb* to grasp its meaning. We can and typically do understand (7) in terms of the decomposition,

(11) $a + R + b$

Nevertheless, if we're to recognise the validity of the inference from (7) to (6), we must be capable of understanding (7) as admitting of the decomposition (9) as well as (11). But do we?

What I'm going to argue is that it's dubious whether we understand the higher-order predicate *aXb* as it occurs in quantified constructions such as (6), as well as atomic ones such as (7). In more general terms, I'll be arguing against the Fregean doctrine that omitting one or more occurrences of a first-level predicate from a sentence is guaranteed to form an intelligible second-level predicate.[11] Then because, as we've seen, DSL requires *aXb* to occur in both (6) and (7) and DSL is a condition of quantifiers having a range, I'll conclude that quantification, so-understood, into the positions of relational predicates is dubious too.

Let's being by considering the semantic significance of *aXb*. With respect to atomic constructions, such as (7), there are powerful considerations in favour of the hypothesis that the significance of right and left-flanking terms is contextually determined by whichever binary predicate occurs between them in a given statement. Suppose, if only for expository purposes, that binary relations are to be conceived, in Williamson's terms, as equipped with two argument places or gaps. Then, as Williamson has argued, we understand each binary predicate as being associated 'with a particular convention as to which flanking names corresponds to which gap' in the binary relation for which the predicate stands (1984: 257). So, for example, we understand 'is before' as associated 'with the convention that the lefthand name denotes what fills the "before" place and the righthand name denotes what fills the "after" place' (1984: 257). As a consequence, we can also understand the converse of 'is before', namely the predicate 'is after', as standing for the very same relation as 'is before'—only that 'is after' comes equipped with the converse convention, *i.e.* that the left-hand name denotes what fills the *after* place whereas the righthand name denotes what fills the *before* place.

Williamson concludes that conceiving of predicates along these lines obviates the need to posit a distinct relation corresponding to each of the

[11] See Frege 1893: §§21-2 and Dummett 1981: 275-6.

mutually converse predicates—one relation will do because the distinct conventions associated with each predicate co-ordinate flanking names with the same gaps but in converse ways. For Williamson the especial benefit of so conceiving predicates is that it avoids the semantic indeterminacy which threatens if we suppose that mutually converse predicates correspond to distinct relations—because of the arising difficulties in determining which relation corresponds to which predicate (1984: 254-5). But there are other benefits too.

Conceiving of predicates along these lines is ontologically parsimonious because it requires fewer relations to be posited, one to a family of mutually converse predicates rather than one for each predicate. This also avoids the need for acknowledging necessary connections between them, the kinds of connections that would obtain if, for example, 'is before' and 'is after' corresponded to distinct relations—because necessarily if some event e_1 is before another e_2, e_2 is after e_1. Associating mutually converse predicates with converse conventions also provides a ready explanation of how language users smoothly negotiate the transition, between, for example, saying that the discovery of Uranus was before the French Revolution and saying that the French Revolution came after the discovery of Uranus. This also explains why mutually converse predicates fail to be intersubstitutable *salva veritate*—why from 'e_1 is before e_2' it doesn't follow that 'e_2 is after e_1' even though 'is before' and 'is after' co-refer. They fail to be intersubstitutable *salva veritate* because mutually converse predicates aren't solely used to specify the relation they denote but also introduce converse conventions about how to interpret the significance of their flanking terms. In the terms of Quine's 'Reference and Modality', converse predicates aren't '*purely referential*' (1953c: 139-140).[12]

Supposing this to be along the right lines then the significance of the higher-order predicate *aXb* as it occurs in atomic constructions, such as (7), is determined by which lower-order predicate occurs between its flanking names. But then *aXb* will lack a uniform significance—because different predicates will be associated with different conventions about how to co-ordinate right and left-flanking names with gaps in their corresponding relations. It also follows that the

[12] See MacBride 2011 for further development of this point.

significance of *aXb* as it occurs in a quantified construction, such as (6), will be left undetermined because in such a construction there occurs no lower-order predicate equipped with a convention to determine the co-ordination of right and left-hand names with gaps in a corresponding relation; there is only a bound variable which ranges over binary relations in general but doesn't pick out a specific one. The upshot is that *aXb* fails to have the self-standing semantic significance which *DSL* requires.

The situation is otherwise with the first-order predicate *xRy* which results from replacing two occurrences of names with variables *x* and *y*. That's because *R* comes equipped with a convention which determines a uniform significance for it independently of which names flank it and this allows for quantifying into their positions.[13] The rule for 'is before' (recall) is that with respect to its denotation and its occurrence in an atomic sentence: the denotation of the left-flanking name fills the *before* slot whilst the denotation of the right-flanking name fills the *after* slot. The rule can be smoothly extended to quantified constructions in which it occurs. For example: the denotation of the left-flanking variable fills the *before* slot whilst the denotation of the right-flanking variable fills the *after* slot upon an assignment of values to variables. By contrast the significance of *aXb* does depend upon which predicate appears in its argument position—because it's the predicate, whichever it be, that's responsible for introducing the convention that determines the significance of the left and right-flanking names. So *aXb* cannot be affirmed of a relation regardless of which predicate denotes it, or regardless of whether any predicate denotes it. *Ergo* the *X* in *aXb* cannot be bound by a quantifier and the result understood in range and entity-invoking terms—whereas the *x* and *y* in *xRy* can.

This argument can be avoided by only permitting quantification into the positions occupied by monadic predicates, *i.e.* where the quantifiers range only over properties but not relations. Consider a monadic second-order predicate *Ya* which results from replacing a monadic predicate (*F* in *Fa*) with a variable *Y*. The

[13] This needn't rule out the possibility that there are context sensitive predicates whose significance varies depending upon the descriptive contents associated with flanking terms. What, however, follows from *DSL* is that quantification into contexts in which such predicates occur cannot be conceived in range and entity-invoking terms.

significance of *Ya* doesn't depend upon whatever predicate fills its argument position to determine the significance of left and right-flanking names, for the simple reason that *Ya* has only one flanking name. But restricting higher-order quantification to monadic higher-order quantification would preclude our taking advantage of several of the envisaged benefits of second-order logic which rely upon higher-order polyadic quantification—for example, enabling us to codify categorical versions of fundamental mathematical principles.[14]

All this is bad news for someone like Williamson. As we have seen, he provides cogent arguments for supposing that mutually converse predicates come equipped with converse conventions for interpreting their flanking names whilst specifying the same relation. He also holds that we should embrace second-order modal comprehension principles, such as $\exists V \Box \forall v_1 \ldots v_n (Vv_1 \ldots Vv_n \leftrightarrow A)$, which involve quantification into the positions of polyadic predicates as well as monadic ones and so depend upon relations as well as properties.[15] But these positions aren't compatible because if mutually converse predicates specify the same relation, then *DSL* prohibits quantifying into their positions.

9. *Intrinsic Order*

The failure of *DSL* in the second-order case can't be forestalled just by positing converse relations, however less than parsimonious that posit otherwise seems, and tolerating whatever semantic indeterminacy subsequently comes our way. It's not enough because to forestall the failure of *DSL* what's required is a uniform convention for interpreting the significance of *aXb* in whatever context it occurs—but just positing converse relations doesn't fulfil this requirement. What's needed is a uniform way of co-ordinating the arrangement of the terms *a* and *b* flanking *X* with the gaps in relations. This can be done if the gaps in relations are intrinsically first and second *etc*. Then *aXb* may be associated with the uniform convention that it is satisfied by a relation *V* just in case the denotation of *a* occupies the first gap and the denotation of *b* the second gap in *V*.

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[14] See Shapiro 1991: 97-109. Consider, for example, Cantor's Theorem understood as the claim that no binary relation can represent the collection of all subsets of its domain ($\forall R\, \exists X\, \forall x\, \exists y[(Rxy\ \&\ \neg Xy) \vee (\neg Rxy\ \&\ Xy)]$).

[15] See Williamson 2013: 227.

Since this condition can be satisfied both by the value of a variable and the denotation of a predicate *R* inserted into argument position of *aXb*, *DSL* would thereby be sustained. But this manner of understanding *aXb* has unappealing consequences. It presupposes that that for any relation there is a fact of the matter about whether, upon an instantiation, an object occupies the first gap of a relation or the second gap *etc.*

The problem here isn't only that there is nothing in the linguistic competence of ordinary speakers which prepares them for conceiving relations as intrinsically ordered—because, for example, an understanding of the statement that Fulvia is to the left of Anthony and an understanding of the statement that Anthony is the husband of Fulvia doesn't presuppose a capacity to keep track of whether Fulvia comes first or second with respect to Anthony in these different relations. The problem is also that this reading of *aXb* entails a global metaphysical hypothesis: that there is an absolute order to the Universe such that all relations can be lined up with the first and second gaps of an arbitrary relation co-ordinated with the first and second gaps of any other relation. [16] It seems extraordinary that this metaphysical hypothesis, which has little else to recommend it, should be tantamount to a higher-order logical principle. The point isn't just that this is an unwholesome commitment but that a hypothesis that isn't logical is being used to secure the validity of what's supposed to be a logical entailment between (7) and (6).[17]

Polyadic higher-order quantification faces another difficulty. In order for (6) to follow from (7), (7) must admit of decomposition (9) as well as (11). But there's a dilemma that makes this uncomfortable. First note that in addition to higher-order claims like (6), there are also higher-order claims of the form

(12) $\exists X\, bXa$

which arise from atomic claims of the form,

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[16] See MacBride 2014 for elaboration of these two points.

[17] This argument against higher-order polyadic quantification can be sidestepped too by only permitting quantification into the positions of monadic predicates. But, again, if we can't have quantification into the positions of polyadic predicates, the case for higher-order logic is correspondingly weakened.

(13) *bRa*

So, by parity of reasoning, recognising the validity of the inference from (13) to (12) requires our recognising that the higher-order predicate *bXa*, which occurs in (12) also occurs in (13). The final piece of the set-up is that (6) follows from (7), and (12) from (13), regardless of whether *R* is symmetric or non-symmetric.

Here's the dilemma. Either *aXb* means something different from *bXa* or they mean the same. If they mean the same, then (7) entails (13). But if *R* is a non-symmetric predicate, then (7) doesn't entail (13) because (7) might be true and (13) false (and *vice versa*). But if they mean something different then even if *R* is a symmetric predicate (7) means something different from (13). But there is nothing mandatory about the view that *a=b* means something different from *b=a* and, in fact, it's questionable whether this is how we naturally understand these statements—it's far from obvious that this view of identity statements reflects the linguistic competence of ordinary speakers. In sum, we cannot recognise the validity of the inference from (7) to (6) or (13) to (12) without generating uncomfortable consequences for either our understanding of non-symmetric predicates or our understanding of symmetric ones.

The first horn of the dilemma is clearly something best avoided. We need to embrace non-symmetric predicates, *i.e.* acknowledge that in some cases *aRb* doesn't entail *bRa*, in order to understand, amongst other things, how mathematical series are generated. But is embracing the second horn really so bad? Can't we, for example, just grant that *a=b* means something different from *b=a* whilst sweetening the pill by adding that they're nevertheless necessarily equivalent statements? Maybe so. But, again, the point is that it doesn't seem the kind of requirement that should be forced upon us by the recognition of the validity of a higher-order entailment.

A concluding reflection. The arguments turning on the *DSL* don't tell us that second-order quantification *per se* is unintelligible. But they do tell us that second-order quantifiers cannot be interpreted along realist lines, as ranging over relations. *Ergo* this provides a reason for interpreting higher-order quantifiers along nominalist lines, whether in substitutional terms or by interpreting quantifiers as a *sui generis* form of operator which although not substitutional don't require to range over anything either.[18] Since the arguments turning on *DSL* have relied upon the assumption that higher-order quantifiers range, *inter alia*, over relations, they may be circumvented by adopting the contrary view that higher-order quantifiers range over the extensions of predicates. But then Quine wouldn't have been far wrong in characterizing higher-order logic as set theory in disguise.[19]

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[18] For a substitutional interpretation of the quantifiers in the service of nominalism see Barcan Marcus (1978). Prior held the view that quantifiers are neither substitutional nor have a range if the corresponding class of instances don't have the function of specifying some entity (Prior 1971: 33-47; MacBride 2006: 444-6). But the arguments from *DSL* only provide partial support for Prior's view, because by Prior's lights, second-order quantifiers only lack a range if first-order predicates don't specify properties or relations.

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